Confidence Interval Solutions

1. **You want to rent an unfurnished one-bedroom apartment in Durham, NC next year. The mean monthly rent for a random sample of 60 apartments advertised on Craig's List (a website that lists apartments for rent) is $1000. Assume a population standard deviation of $200. Construct a 95% confidence interval.**

$$\left(\bar{x} - Z^* \frac{\sigma}{\sqrt{n}}, \bar{x} + Z^* \frac{\sigma}{\sqrt{n}}\right)$$

($1000 – 1.96*$200/sqrt(60), $1000 + 1.96*$200/sqrt(60))

($949.39, $1050.61)

We are 95% confident that the interval ($949.39, $1050.61) covers the true mean monthly rent of Durham apartments listed on Craig's List.

2. **To what population of apartments can you appropriately infer from your sample in #1?**

We can most accurately infer to Durham apartments listed on Craig's List. We should not infer to all apartments in Durham because we do not know if apartments on Craig's List are representative of all apartments in Durham.

3. **How large a sample of one-bedroom apartments above would be needed to estimate the population mean within plus or minus $50 with 90% confidence?**

The z multiplier for a 90% confidence interval is 1.645. We want z*σ/sqrt(n) to equal $50.

$$\$50 = z * \frac{\sigma}{\sqrt{n}} = \frac{1.645 * 200}{\sqrt{n}} = \frac{329}{\sqrt{n}}$$

Solve for n. sqrt(n) = 329/50 = 6.58. Square both sides and you get an n equal to 43.3. To calculate a margin of error +/- $50, you would need to randomly sample 44 rental apartments on Craig's List.

4. **Duncan Jones kept careful records of the fuel efficiency of his car. After the first 100 times he filled up the tank, he found the mean was 23.4 miles per gallon (mpg) with a population standard deviation of 0.9 mpg. Compute the 95 percent confidence interval for his mpg.**

$$(\bar{x} - Z^* \frac{\sigma}{\sqrt{n}}, \bar{x} + Z^* \frac{\sigma}{\sqrt{n}})$$

(23.4 – 1.96*0.9/sqrt(100), 23.4 + 1.96*0.9/sqrt(100))

We are 95% confident that the interval (23.22 mpg, 23.58 mpg) includes the true mean mpg of Duncan's automobile.

5. **Which of the assumptions listed above might be problematic in making inference to the population in Question 4?**

The data likely do not meet the independence assumption. Most likely there is serial correlation in the observed mpg (patterning of mpg through time because of car age, serially correlated weather, etc.). A confidence interval is probably not the appropriate tool to make inferences about the true mean mpg.

6. **True or False: The population mean (μ) is a random variable that will fall within a confidence interval with 95% probability (with repeated sampling).**

FALSE. The population mean is NOT a random variable but a population parameter. The population parameter has one true value (it does not shift). The confidence interval shifts based on the random sampling process. With repeated sampling, 95% of the confidence intervals will include the true population mean.

7. **True or False: With all else constant, an increase in population standard deviation will shorten the length of a confidence interval.**

FALSE. With all else constant, increasing the population standard deviation will lengthen the confidence interval.