Supplementary Material: Hydraulic Model

Model Development

Consider a mass of water in the height interval (*b,u*) that is recharged across lower boundary *b* with water supplied from roots and is withdrawn with evaporative demand across upper boundary *u*. Based on observations of flux at (*b,u*) we wish to estimate the daily stored water use represented by the interval (*b,u*). Change in stored water *W* (mass or volume concentration) depends on the gradient in flux:

$$\frac{\partial W(h,t)}{\partial t} = -\frac{\partial Q(h,t)}{\partial h} \qquad \text{(Eq. S1)}$$

Focusing on a specific height (*h*) interval (*b,u*) of interest for tree *i* in treatment *j* we have:

$$\frac{dW_{bu(ij,t)}}{dt} = -\left[Q_{u(ij,t)} - Q_{b(ij,t)}\right] \qquad \text{(Eq. S2)}$$

Where evaporative demand on *W* over (*b,u*) for individual *i* in treatment *j* at time *t* (g m^{-2} basal sapwood area) is imposed by a flux across *h* = *u* and transferred to *h* = *b* by the accumulated deficit, a negative xylem pressure, whenever flux at *u* exceeds that at *b*. When evaporative demand is high, the first flux term exceeds the second, water is withdrawn, and negative xylem pressure develops within (*b,u*). As evaporative demand at *u* declines below recharge across *b*, water mass increases, and xylem pressures become less negative.

If flux at the lower sensor is proportional to the deficit, then:

$$Q_{b(ij,t)} = \alpha_{(ij,t)}\left(\omega_{bu(ij)} - W_{bu(ij,t)}\right) \qquad \text{(Eq. S3)}$$

where $\omega_{bu(ij)}$ is the water capacity of xylem for the height interval (*b,u*) and α is a rate constant (min^{-1}). The reciprocal of α is the time constant (κ, min) discussed in the main text. The discrete time approximation of Eq. S2 is:

$$W_{ub(ij,t+dt)} - W_{bu(ij,t)} = \left(Q_{b(ij,t)} - Q_{u(ij,t)}\right)dt \qquad \text{(Eq. S4)}$$

If we let:

$$D_{bu(ij,t)} = \omega_{bu(ij,t)} - W_{bu(ij,t)} \quad \text{(Eq. S5)}$$

where $D_{bu(ij)}$ is the deficit of water, then we have:

$$\frac{D_{bu(ij,t+dt)} - D_{bu(ij,t)}}{dt} = \left(Q_{u(ij,t)} - Q_{b(ij,t)}\right) \quad \text{(Eq. S6)}$$

Note that with large α (Eq. S3) a deficit cannot accumulate. Parameter α translates the height gradient in flux to change in flux at the boundary *b*:

$$\frac{Q_{b(ij,t+dt)} - Q_{b(ij,t)}}{dt} = \alpha_{(ij,t)}\left(Q_{u(ij,t)} - Q_{b(ij,t)}\right) \quad \text{(Eq. S7)}$$

Inference

Estimates of α must accommodate observation error in the flow rates at $h = u$ and $h = b$. Let the observation error for flux $Q_{ij,t}$ be:

$$Q_{h(ij,t)} \sim N\left(\tilde{Q}_{h(ij,t)}, \sigma^2\right) \quad \text{(Eq. S8)}$$

where $\tilde{Q}_{ij,t}$ is the true flux. We assume that the error variance is similar at the two heights. To simplify notation, let:

$$y_{(ij,t)} = \frac{Q_{b(ij,t+dt)} - Q_{b(ij,t)}}{dt} \quad \text{(Eq. S9)}$$

be change in flux at *b* and

$$x_{(ij,t)} = Q_{u(ij,t)} - Q_{b(ij,t)} \quad \text{(Eq. S10)}$$

be the difference in flux at the two heights. Then the model for inference on α is:

$$N\left(y_{(ij,t)} \middle| \alpha_{(ij,t)} x_{(ij,t)}, 2\sigma^2\left(dt^{-2} + {\alpha_{(ij,t)}}^2\right)\right) \quad \text{(Eq. S11)}$$

We assume the priors:

$$N\left(\alpha_{(ij,t)} \middle| 0.1, 10\right) \quad \text{(Eq. S12)}$$
$$IG(\sigma^2 | s_1, s_2) \quad \text{(Eq. S13)}$$

where s_1 is the number of days included in the estimate, $s_2 = m_s(s_1 - 1)$, and $m_s = (0.2 \times 4)^2$ assumes that the error is approximately 20% of the flux, which averages near 4 g m^{-2} s^{-1}. The number of observations far exceeds the number of days included in the analysis, thus making this prior weak.

Treatment effects (eCO_2 and N_F) were incorporated in the recharge rate (α), as was that of volumetric soil moisture ($M_{(t)}$). The models for inference on these effects were of the form:

$$N\left(y_{(ij,t)}\middle|\mathbf{z}_{(\boldsymbol{ij},\boldsymbol{t})}^{T}\mathbf{a}x_{(ij,t)}, 2\sigma^2\left(dt^{-2}+\left(\mathbf{z}_{(\boldsymbol{ij},\boldsymbol{t})}^{T}\mathbf{a}\right)^2\right)\right) \quad \text{(Eq. S14)}$$

where $\mathbf{z}_{(\mathbf{ij,t})}$ is a length-p design vector, containing treatments and $M_{(t)}$ as main effects and interactions and **a** is the length-p vector of coefficients. The minimal model contains only an intercept, and the maximum model contains all main effects, two-, and three-way interactions. The number of trees and half-hourly observations for the calibration period is given in Table S2. The number of sensors in each treatment for each year during the evaluation study period is given in Table S3.

Models were compared with the Bayesian information criterion (BIC) based on the number of observations (n) and parameters (p):

$$BIC = n\ln(2\hat{\sigma}^2) + p\ln(n) + \sum_{ij,t}\ln\left(dt^{-2}+\left(\mathbf{z}_{(\boldsymbol{ij},\boldsymbol{t})}^{T}\hat{\mathbf{a}}\right)^2\right) \quad \text{(Eq. S15)}$$

where posterior means are indicated with hats. The mean parameters and difference in BIC for each model are given in Table S4.

Table S1. Abbreviations found only in Supplementary Material.

D_{bu}	Stored water deficit of height interval (*b*,*u*) per unit sapwood area at breast height
i	Index for sensor pair (breast-height and base of crown)
j	Index for treatment
s_1, s_2	Rate and scale of inverse-gamma distribution for prior of σ^2
W	Stored water content per unit sapwood area at breast height
x	Difference in flow *Q* at upper and lower sensors
y	Rate of change in Q^b from time *t* to *t*+1
z	Design vector for given model
α	Recharge rate constant across lower boundary *b*
σ^2	Error variance of Q^h observations
ω	Stored water capacity

Table S2. Trees sampled for calibration of hydraulic model in 2009 by treatment.

Treatment	**Trees**	***n***	r_M	r_I	***DBH***	***HBC***
AC	6	11802	0.55	0.30	21.3 (6.0)	10.6 (1.3)
EC	7	20545	0.46	0.36	21.1 (4.3)	11.7 (1.6)
AF	7	12441	0.71	0.26	22.4 (5.3)	10.8 (2.0)
EF	6	18880	0.85	0.69	24.3 (6.2)	10.8 (1.8)

n: the number of half-hourly sap flux observations at two heights, *DBH*: mean and standard deviation of diameter at base height (cm) and *HBC*: height to base of crown (m). Treatments: AC = ambient CO_2 unfertilized, EC = elevated CO_2 unfertilized, AF = ambient CO_2 fertilized, EF = elevated CO_2 unfertilized.

Table S3. Number of sap flux sensors by treatment, year and species for evaluation of stomatal conductance model.

Year	**Depth (mm)**	**Ambient CO_2**		**Elevated CO_2**	
		AC	**AF**	**EC**	**EF**
1998	0-20	40		49	
	20-40	6		6	
1999 - 2000	0-20	35		38	
	20-40	6		6	
2001 - 2004	0-20	39		39	
	20-40	17		17	
	40-60	11		11	
2005 - 2007	0-20	25	30	25	29
	20-40	9	12	13	13
	40-60	7	9	6	10
2008	0-20	23	25	24	25
	20-40	9	17	12	10
	40-60	6	6	6	6

Treatments: AC = ambient CO_2 unfertilized, EC = elevated CO_2 unfertilized, AF = ambient CO_2 fertilized, EF = elevated CO_2 unfertilized.

Table S4. Models for the recharge rate constant (α, min^{-1}).

Int	**eCO_2**	**N_F**	**eCO_2 $\times N_F$**	**$M_{(t)}$**	**eCO_2 $\times M_{(t)}$**	**N_F $\times M_{(t)}$**	**$eCO_2 \times N_F$ $\times M_{(t)}$**	**ΔBIC**
0.0276	0.0026	-0.0009	-0.0065	-0.0631		0.0251		0
0.0278	0.0022	-0.0010	-0.0064	-0.0637	0.0017	0.0255		24.2
0.0259	0.0071	0.0028	-0.0194	-0.0554	-0.0183	0.0082	0.0571	49.8
0.0253	0.0021	0.0047	-0.0061	-0.0525				58.7
0.0250	0.0029	0.0046	-0.0062	-0.0514	-0.0033			122.7
0.0272	-0.0007			-0.0507				353.6
0.0271	-0.0004			-0.0502	-0.0014			404.7
0.0272				-0.0517				675.8
0.0162	-0.0021							781.4
0.0285	-0.0024	-0.0026		-0.0628	0.0106	0.0236		818.0
0.0258	-0.0012	0.0027		-0.0507	0.0044			903.4
0.0275		-0.0020		-0.0576		0.0207		925.3
0.0255	-0.0002	0.0027		-0.0492				960.1
0.0140	-0.0002	0.0044	-0.0035					964.8
0.0275	0.0001	-0.0021		-0.0578		0.0209		965.4
0.0255		0.0027		-0.0494				1034.5
0.0146	-0.0015	0.0032						1463.8
0.0155								1694.7
0.0139		0.0035						2131.1

Int= intercept, eCO_2 = elevated atmospheric carbon dioxide, N_F = nitrogen fertilization, $M_{(t)}$ = volumetric soil moisture. ΔBIC: Bayes information criteria difference to the lowest BIC model.