

Math Review – practice questions
Some sourced from <http://tutorial.math.lamar.edu>, others original
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1. Simultaneous equations. Solve for x and y:

a.
$$\begin{aligned} 3x - y &= 7 \\ 2x + 3y &= 1 \end{aligned}$$

b.
$$\begin{aligned} x - 2y + 3z &= 7 \\ 2x + y + z &= 4 \\ -3x + 2y - 2z &= -10 \end{aligned}$$

c.
$$\begin{aligned} 2x + 5y &= -1 \\ -10x - 25y &= 5 \end{aligned}$$

2. True/false. The following are functions. Why or why not?

a. $x^2 + y^2 = 4$

b.
$$f(x) = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$$

3. Differentiate each of the following functions:

a. $f(x) = (6x^3 - x)(10 - 20x)$

b. $g(x) = 3e^x + e^{3x}$

c. $g(x) = (6x^3 - x - 4)^2$

d. $W(z) = \frac{3z^2 + 9}{2 - z}$

e. $f(x) = e^x + \ln x$

f. $f(x) = e^x + 10x^3 \ln x$

g. $y(x) = \sqrt[3]{x^2}(2x - x^2)$

4. Find all first and second order partial derivatives (including cross-partials) for:

a. $f(x, y) = x^4 + 6\sqrt{y} - 10$

b. $f(x, y) = 3x^2 + 2xy - 4x^{-1}y^2$

5. Find the total derivatives of the following functions:

a. $f(x, y) = x^2 + xy + y^2$

b. $f(x, y, z) = \ln x + yz + x^2y^2z^2$

6. Identify all minima, maxima, and inflection points of the following functions:

a. $f(x) = 2x^2 + 6x + 7$

b. $g(x) = -x^2 - 4x + 4$

7. Find the x and y intercepts of $y = 3x - 6$. (The intercepts are where the function intersects the x and y axes.) What are the (x,y) coordinates at these intercepts?

8. Find all values of x such that these two functions have the same slope. Do the two functions touch at this point? How do you know?

$$\begin{aligned}f(x) &= 3x - 1 \\g(x) &= x^2 - 4\end{aligned}$$

9. Evaluate the following integrals if possible:

a.

$$\int x^4 + 3x - 9 \, dx$$

b.

$$\int_{-1}^2 y^2 + 2y \, dy$$

c.

$$\int_{-3}^1 6x^2 - 5x + 2 \, dx$$

10. The growth of a colony of bacteria is given by the equation:

$$Q = Q_0 e^{0.195t}$$

If there are initially 500 bacteria present ($Q_0 = 500$) and t is given in hours, determine each of the following.

a. How many bacteria are there after half a day?

b. How long will it take before there are 10,000 bacteria in the colony?

c. For a different species of bacteria, the growth of a colony is given by the equation $Q = Q_0 e^{0.5t}$. If there are initially 500 bacteria present ($Q_0 = 500$) and t is given in hours, would you expect this species to reach 10000 bacteria in the colony in more time, less time, or about the same amount of time as the species in which $Q = Q_0 e^{0.195t}$?

11. After Ella finishes her MEM at Duke, she receives a job offer with a \$15,000 signing bonus. She puts the entire bonus into an account that earns interest at a rate of 6 percent (annually) for 5 years. Assuming she does not make any further deposits or withdrawals, determine how much money will be in the account at the end of 5 years, if interest is:

- a. Compounded annually
- b. Compounded monthly
- c. Compounded continuously

Suggestions for additional exercises:

- The final slide in the PowerPoint presentation provides several resources for additional explanations and problems.
- Another good resource (that provided the source for some of these problems) is <http://tutorial.math.lamar.edu/>.
- The Wolfram Alpha search engine is a great resource for plotting functions and calculating algebraic and numerical answers. You can make up your own problems and check your answers with this search engine. For instance, the answer to 9b (with a graph!) is here: <https://www.wolframalpha.com/input/?i=integral+of+y%5E2%2B2y+dy+from+-1+to+2>.