1. Simultaneous equations. Solve for \( x \) and \( y \):

a. \[ \begin{align*}
3x - y &= 7 \\
2x + 3y &= 1
\end{align*} \]

\[ \text{Solve by substitution:} \]
\[ \begin{align*}
3x - 2 &= y \\
2x + 3(3x - 2) &= 1 \\
2x + 9x - 6 &= 1 \\
11x &= 7 \\
x &= \frac{7}{11} \\
y &= -1
\]

b. \[ \begin{align*}
2x + 5y &= -1 \\
-10x - 25y &= 5
\end{align*} \]

\[ \text{Solve by substitution:} \]
\[ \begin{align*}
2x + 1 &= 5y \\
10x + 5 &= -25y
\end{align*} \]
\[ \begin{align*}
-10x + (10x + 5) &= 5 \\
-10x + 10x + 5 &= 5 \\
5 &= 5
\end{align*} \]
\[ \text{Any point on } y = -\frac{2}{5}x - \frac{1}{5} \text{ is a solution.} \]

c. \[ \begin{align*}
x - 2y + 3z &= 7 \\
2x + y + z &= 4 \\
-3x + 2y - 2z &= -10
\end{align*} \]

\[ \text{Solve by elimination:} \]
\[ \begin{align*}
x - 2y + 3z &= 7 \\
4x + 2y + 2z &= 8 \\
-3x + 2y - 2z &= -10
\end{align*} \]
\[ \Rightarrow \]
\[ \begin{align*}
x - 2y + 3z &= 7 \\
x + 3z &= 15 \\
-2x + z &= -3
\end{align*} \]
\[ \Rightarrow \]
\[ \begin{align*}
x - 2y + 3z &= 7 \\
5x + 5z &= 15 \\
15x &= 30
\end{align*} \]
\[ \Rightarrow \]
\[ \begin{align*}
x &= \frac{2}{5} \\
5(2) + \frac{5}{2} &= 15 \\
2z &= 15 \\
2 - 2y + z &= 7 \\
y &= -1
\end{align*} \]
2. True/false: The following are functions (why/why not?)
   a.
   \[ x^2 + y^2 = 4 \]
   Note that \( x^2 - 4 - y^2 \)
   If \( x = 0 \), \( y^2 = 4 \) \( \Rightarrow y = \pm 2 \).
   Then there is not a unique \( y \) for each \( x \). Not a function.

   b. \[ f(x) = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases} \]
   This is a piecewise function. There is a unique \( f(x) \) for each \( x \).
   Note that although it is continuous, it is not a continuously differentiable function.

3. Differentiate each of the following functions:
   a. \[ y(x) = \sqrt[3]{x^2(2x-x^2)} \]
   \[ y'(x) = 3x \frac{1}{3} (2x-x^2)^{1/3} + x^2 \frac{1}{3} (2-2x) = y'(x) \]
   Product rule: \[ \frac{1}{3} x(2x-x^2) \]

   b. \[ f(x) = (6x^3-x)(10-20x) \]
   Product rule: \[ f'(x) = (18x^2-1)(10-20x) + (6x^3-x)(-20) \]

Most questions sourced from http://tutorial.math.lamar.edu/, compiled by David Kaczan
4. Find all first and second order partial derivatives for the following functions:

   a. 
   \[
   h(s, t) = t^2 \ln(s^2) + \frac{9}{t^3} - 7\sqrt{s^4}
   \]
   \[
   \frac{dh}{ds} = \frac{2t^2}{s} - \frac{4t}{s^2} \\
   \frac{dh}{dt} = 2t \ln(s^2) - \frac{27}{t^4}
   \]
   \[
   \frac{d^2 h}{d s d s} = \frac{4t^2}{s^3} - \frac{12t}{s^4}
   \]
   \[
   \frac{d^2 h}{d s d t} = 4t \ln(s^2) + \left( \frac{4}{t^2} \right)
   \]
   \[
   \frac{d^2 h}{d t d t} = \frac{14t^6}{s^4}
   \]

   b. 
   \[
   f(x, y) = x^4 + 6\sqrt{y} - 10
   \]
   \[
   \frac{df}{dx} = 4x^3 \\
   \frac{df}{dx^2} = 12x^2
   \]
   \[
   \frac{df}{dy} = \frac{3}{\sqrt{y}} \\
   \frac{d^2 f}{dy^2} = \left( -\frac{3}{2} \right) y^{-3/2} = \frac{3}{2\sqrt{y}^3}
   \]
   \[
   \frac{d^2 f}{dx dy} = \frac{d^2 f}{dy dx} = 0
   \]
c. \[ W(z) = \frac{3z + 9}{2 - z} \]

Quotient Rule: \[ \frac{W'(z)}{W(z)} = \frac{3(2-z) - 1(3z+9)}{(2-z)^2} = \frac{6 - 3z - 3z - 9}{(2-z)^2} = \frac{18}{(2-z)^2} \]

\( h(x) = \frac{4\sqrt{x}}{x^2 - 2} \)

\[ h'(x) = \frac{4\sqrt{x}}{x^2 - 2} \cdot \left( \frac{1}{2} \right) \frac{1}{2x} = \frac{2x - \sqrt{x}}{(x^2 - 2)^2} = \frac{-6x^3 - 4x^4}{(x^2 - 2)^2} \]

\( R(z) = \sqrt{5z - 8} \)

\[ k'(z) = (5z - 8)^{1/2} \]

Chain Rule: \[ \frac{1}{2} (5z - 8)^{1/2} \cdot (5) = \frac{5}{2\sqrt{5z - 8}} = R'(z) \]

\( g(x) = \ln(x^4 + x^4) \)

Chain Rule: \[ g'(x) = \frac{-4x^{-2} + 4x^3}{x^4 + x^4} \]

g. \[ f(x) = 3e^x + 10x^2 \ln x \]

Quotient Rule: \[ 3e^x + \left[ 10 \cdot \frac{1}{x} \right] x^2 \ln x + 10x^2 \left( \frac{1}{x} \right) \]

\[ = 3e^x + 30x^1 \ln x + 10x^1 = f'(x) \]

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5. Find the total derivatives of the following functions:
   a. \( f(x, y) = x^2 + yx + y^2 \)

   \[
   d\ell = \frac{\partial f}{\partial x} \, dx + \frac{\partial f}{\partial y} \, dy
   \]

   \[
   = (2x + y) \, dx + (x + 2y) \, dy
   \]

   b. \( f(x, y) = \ln(x) + yz + x^2y^2z^2 \)

   \[
   d\ell = \left( \frac{1}{x} + 2xyz^2 \right) \, dx + (2xz + 2y^2z^3) \, dy + (y + 2xz^3y^2) \, dz
   \]

6. Characterize all extreme points of the following functions:
   a. \( f(x) = 2x^2 + 6x + 7 \)

   \( \ell'(x) = 4x + 6 \)

   solve \( \ell'(x) = 0 \):
   \[
   4x + 6 = 0 \implies x = -\frac{3}{2}
   \]

   Note: \( \ell''(x) = 4 > 0 \) at \(-\frac{3}{2}, \frac{5}{2}\) is a minimum.
b. 

\[ f(x) = \frac{1}{2}x^4 - 3x^3 - 9x^2 + 6 \]

\[ f'(x) = 0 \Rightarrow 2x^3 - 9x^2 - 18x \]

\[ = x(2x^2 - 9x - 18) \]

\[ = x(2x + 3)(x - 6) \]

\[ \Rightarrow x = -\frac{3}{2}, \quad x = 6, \quad x = 0 \]

Critical points: \((-\frac{3}{2}, -\frac{3}{2}), (6, -318), (0, 6)\)

Hence \((6, -318)\) is the global minimum.

7. Find the x and y intercepts:

\[ f(x) = x^2 - x + 6 \]

x-intercepts: \(0 = x^2 - x - 6\)

\[ 0 = (x - 3)(x + 2) \]

\[ x = 3, \quad x = -2 \]

y-intercept: \(f(0) = -6\)

8. Evaluate the following integrals if possible:

a.

\[ \int x^4 + 3x - 9 \, dx \]

\[ = \frac{1}{5}x^5 + \frac{3}{2}x^2 - 9x + c \]
b. 
\[ \int \frac{3}{x^{3/2}} + \frac{7}{x^{5}} + \frac{1}{6\sqrt{x}} \, dx \]
\[ = \int \frac{3}{x^{1/2}} + \frac{7}{x^{5}} + \frac{1}{6x^{1/2}} \, dx \]
\[ = \left( \frac{4}{7} \right) x^{1/2} - \frac{7}{4} x^{-4} + \left( \frac{1}{6} \right) (2) x^{1/2} + C \]

c. 
\[ \int 5t^3 - 10t^{-6} + 4 \, dx \]
\[ = \frac{5}{4} t^4 + \frac{10}{5} t^{-5} + 4t + C \]

d. 
\[ \int_{-1}^{2} y^2 + y^{-2} \, dy \]
Note that this cannot be evaluated as \( t = 0 \), which is in the domain of integration. No solution.

e. 
\[ \int_{-3}^{1} 6x^2 - 5x + 2 \, dx \]
\[ = \left[ 2x^3 - \frac{5}{2} x^2 + 2x \right]_{-3}^{1} = \left( 2 - \frac{5}{2} + 2 \right) - \left( -\frac{5}{2} - 6 \right) = 84 \]
\[ \int_{-2}^{3} 5t^6 - 10t + \frac{1}{t} \, dt \]

No solution at \( t = 0 \), which is in the integral's domain.

9. The growth of a colony of bacteria is given by the equation:

\[ Q = Q_0 e^{0.195t} \]

If there are initially 500 bacteria present and \( t \) is given in hours determine each of the following.

a. How many bacteria are there after a half of a day?

\[ Q(t = 0) = 500 = Q_0 e^{0.195(0)} \]
\[ \therefore 500 = Q_0 \]
\[ \therefore Q = 500 e^{0.195t} \]

At \( t = 12 \),
\[ Q(12) = 500 e^{0.195(12)} \]
\[ \approx 8190.6 \]

b. How long will it take before there are 10000 bacteria in the colony?

\[ Q(t) = 10000 = 500 e^{0.195t} \]
\[ \therefore e^{0.195t} = 20 \]
\[ \therefore 0.195t = \ln(20) \]
\[ \therefore t = 15.36 \]

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10. We are going to invest $100,000 in an account that earns interest at a rate of 7.5 percent (annually) for 54 months. Determine how much money will be in the account if interest is:

a. Compounded monthly

\[ 100,000 \left(1 + \frac{0.075}{12}\right)^{54} \approx 139,996.84 \]

b. Compounded continuously

\[ 100,000 \cdot e^{0.075 \cdot \frac{54}{12}} \approx 140,143.96 \]