Comparison of Means Solutions

1. True or False: A p-value is the probability that the null hypothesis is true.

FALSE. A p-value is a conditional probability—given the null hypothesis is true, it's the probability of getting a test statistic as extreme or more extreme than the calculated test statistic.

2. True or False: A very small p-value (<0.01) provides evidence in support of the null hypothesis.

FALSE. A very small p-value provides evidence AGAINST the null and in support of the alternative hypothesis.

3. True or False: All else constant, in a one sample test, the greater the sample size, the greater the (positive) test statistic or the smaller the (negative) test statistic.

TRUE. All else constant, the larger the sample size, a positive test statistic will increase in size while a negative test statistic will become more negative.

$$\text{z-statistic} = \frac{\bar{x} - \mu_o}{\sigma/\sqrt{n}}$$

4. We want to examine the effectiveness of an environmental education seminar. We randomly select 50 seminar attendees and give them a test on environmental topics both before and after the seminar. We want to determine if the environmental education seminar improved environmental understanding. Which of the following is the most appropriate test?

    a. a paired, two-sided test

    b. a two independent sample, one-sided test

    c. a paired, one-sided test

5. We want to determine whether a certain make and model of car has a greater fuel efficiency in miles per gallon (mpg) than 50mpg. We randomly sample 100 cars of the same make and model. The mean mpg of the sample is 53 mpg and the sample standard deviation is 5 mpg.

a. Establish the null and alternative hypotheses.

$$\text{Ho: } \mu \leq 50 \text{ mpg}$$
$$\text{Ha: } \mu > 50 \text{ mpg}$$

b. Calculate the appropriate test statistic.

This is a one-sided t-test. We will use the t-distribution because we are estimating the population standard deviation from the sample standard deviation.

$$\text{t-statistic} = \frac{\bar{x} - \mu_o}{s/\sqrt{n}}$$

$$t = (53 - 50)/(5/\text{sqrt}(100))$$

$$t = 3/(0.5) = 6$$

c. True or False: We should calculate a z-statistic.

False. See above.

d. Calculate the p-value and compare to an alpha level of 0.05 to draw your conclusions.

By looking up the p-value on a t-distribution table, with 99 degrees of freedom (will probably need to look up 100 df), we find a p-value $p(t>6)<0.0005$. This p-value provides strong evidence against the null (we can say we reject the null hypothesis) and in support of the alternative hypothesis. The evidence strongly suggests that fuel efficiency, as measured in mpg of this automobile make and model is greater than 50 mpg. A student t-table can be found here at the Texas A & M's <u>statistics website</u>.

6. We sample two forests (n=100 in each forest) of loblolly pine trees and measure their diameter at breast height (dbh) in centimeters. We establish the following hypotheses:

Ho: The mean DBH measurement of loblolly trees in forest A is less than or equal to the mean in forest B.

Ha: The mean of DBH measurement of loblolly trees in forest A is greater than the mean in forest B.

We use the following test statistic (Welch's t-test) equation and get a t-statistic of -3.6.

$$\text{t-stat} = \frac{(\bar{x}_A - \bar{x}_B) - (\mu_A - \mu_B)}{\sqrt{\frac{s_A^2}{n_A} + \frac{s_B^2}{n_B}}}$$

True or False: This test statistic provides strong evidence against the null hypothesis.

FALSE. In our alternative hypothesis, we proposed that the mean DBH of trees in forest A are greater than forest B. However, we get a (highly) negative t-stat. To get a negative t-stat, the mean of sample A must be less than the mean of sample B.