Supplementary Methods

Contents

Derivation of whole-tree water use

Whole tree water use can be calculated by integrating Equation 1 from the main text. Here we present solutions to these integrals for each model. Parameter definitions follow the main text and $\gamma(s,x)$ is the incomplete gamma function: $\gamma(s,x) = \int_0^x t^{s-1}e^{-t}dt$.

Half-Gaussian:

$$Q = 2\pi \int_0^R (R-x)f(x)dx$$
$$= 2\pi J(\int_0^R Re^{-\beta x^2}dx - \int_0^R xe^{-\beta x^2}dx)$$

With substitution of $u = \beta x^2$ (and $du = 2\beta x dx$),

$$= 2\pi J(R\int_0^{\beta R^2} \frac{1}{2\beta x}e^{-u}du - \int_0^{\beta R^2} x\frac{1}{2\beta x}e^{-u}du)$$
$$= 2\pi J(R\int_0^{\beta R^2} \frac{1}{2\beta u^{1/2}\beta^{-1/2}}e^{-u}du - \frac{1}{2\beta}\int_0^{\beta R^2} e^{-u}du)$$
$$= 2\pi J(\frac{R}{2\sqrt{\beta}}\int_0^{\beta R^2} u^{-1/2+1-1}e^{-u}du - \frac{1}{2\beta}\int_0^{\beta R^2} e^{-u}du)$$
$$= 2\pi J(\frac{R}{2\sqrt{\beta}}\gamma(1/2,\beta R^2) - \frac{1}{2\beta}\gamma(1,\beta R^2))$$

Gaussian:

$$Q = 2\pi \int_0^R (R-x)f(x)dx$$
$$= 2\pi J(\int_0^R Re^{-\beta(x-\alpha)^2}dx - \int_0^R xe^{-\beta(x-\alpha)^2}dx)$$
$$= 2\pi J(\int_0^R Re^{-\beta(x-\alpha)^2}dx - \int_0^R (x-\alpha)e^{-\beta(x-\alpha)^2}dx + \int_0^R \alpha e^{-\beta(x-\alpha)^2}dx)$$

With substitution of $u = \beta(x-\alpha)^2$ (and $du = 2\beta(x-\alpha)dx$),

$$= 2\pi J(R\int_{\beta\alpha^2}^{\beta(R-\alpha)^2} \frac{1}{(2\beta(x-\alpha))}e^{-u}du - \int_{\beta\alpha^2}^{\beta(R-\alpha)^2} \frac{(x-\alpha)}{(2\beta(x-\alpha))}e^{-u}du + \alpha\int_{\beta\alpha^2}^{\beta(R-\alpha)^2} \frac{1}{(2\beta(x-\alpha))}e^{-u}du)$$
$$= 2\pi J(R\int_{\beta\alpha^2}^{\beta(R-\alpha)^2} \frac{1}{(2\beta(\alpha+u^{1/2}\beta^{-1/2}-\alpha))}e^{-u}du - \frac{1}{2\beta}\int_{\beta\alpha^2}^{\beta(R-\alpha)^2} e^{-u}du + \alpha\int_{\beta\alpha^2}^{\beta(R-\alpha)^2} \frac{1}{(2\beta(\alpha+u^{1/2}\beta^{-1/2}-\alpha))}e^{-u}du)$$
$$= 2\pi J(\frac{R}{2\sqrt{\beta}}\int_{\beta\alpha^2}^{\beta(R-\alpha)^2} u^{-1/2}e^{-u}du - \frac{1}{2\beta}\int_{\beta\alpha^2}^{\beta(R-\alpha)^2} e^{-u}du + \frac{\alpha}{2\sqrt{\beta}}\int_{\beta\alpha^2}^{\beta(R-\alpha)^2} u^{-1/2}e^{-u}du)$$

$$= 2\pi J(\frac{R}{2\sqrt{\beta}}(\gamma(1/2,\beta(R-\alpha)^2)+\gamma(1/2,\beta\alpha^2))-\frac{1}{2\beta}(\gamma(1,\beta(R-\alpha)^2)+\gamma(1,\beta\alpha^2))+\frac{\alpha}{2\sqrt{\beta}}(\gamma(1/2,\beta(R-\alpha)^2)+\gamma(1/2,\beta\alpha^2))$$

$$= \frac{\pi J}{\sqrt{\beta}}(R(\gamma(1/2,\beta(R-\alpha)^2)+\gamma(1/2,\beta\alpha^2))-\frac{1}{\sqrt{\beta}}(\gamma(1,\beta(R-\alpha)^2)+\gamma(1,\beta\alpha^2))+\alpha(\gamma(1/2,\beta(R-\alpha)^2)+\gamma(1/2,\beta\alpha^2))$$

Gamma:

$$Q = 2\pi \int_0^R (R-x)f(x)dx$$
$$= 2\pi J(\int_0^R R(\beta x)^\alpha e^{-\beta x}dx - \int_0^R x(\beta x)^\alpha e^{-\beta x}dx)$$

With substitution of $u = \beta x$ (and $du = \beta dx$),

$$= 2\pi J(\int_0^{\beta R} R u^\alpha e^{-u}\beta^{-1}du - \int_0^{\beta R} u\beta^{-1}u^\alpha e^{-u}\beta^{-1}du)$$
$$= 2\pi J(\frac{R}{\beta}\int_0^{\beta R} u^{\alpha+1-1}e^{-u}du - \frac{1}{\beta^2}\int_0^{\beta R} u^{\alpha+2-1}e^{-u}du)$$
$$= 2\pi J(\frac{R}{\beta}\gamma(\alpha+1,\beta R) - \frac{1}{\beta^2}\gamma(\alpha+2,\beta R))$$

Beta:

$$Q = 2\pi R^2 \int_0^1 (1-r)f(r)dr$$
$$= 2\pi R^2 J(\int_0^1 r^{\alpha-1}(1-r)^{\beta-1}dr - \int_0^1 r r^{\alpha-1}(1-r)^{\beta-1}dr)$$
$$= 2\pi R^2 J(\int_0^1 r^{\alpha-1}(1-r)^{\beta-1}dr - \int_0^1 r^{\alpha+1-1}(1-r)^{\beta-1}dr)$$
$$= 2\pi R^2 J(B(\alpha,\beta) - B(\alpha+1,\beta))$$

Predictions for new observations

The correction factor for scaling indpendent observations (Equation 3 in main text) is based on the ratio of the whole tree flow and the flow in the measured portion

$$c = Q_{all}/Q_{meas}$$

For the Gamma model, this is

$$c = \frac{2\pi J\int_0^R (R-x)(\beta x)^\alpha e^{-\beta x}dx}{2\pi J\int_a^b (R-x)(\beta x)^\alpha e^{-\beta x}dx}$$

$$= \frac{\int_0^R (R-x)(\beta x)^\alpha e^{-\beta x}dx}{\int_0^b (R-x)(\beta x)^\alpha e^{-\beta x}dx - \int_0^a (R-x)(\beta x)^\alpha e^{-\beta x}dx}$$

$$= \frac{\frac{R}{\beta}\gamma(\alpha+1,\beta R) - \frac{1}{\beta^2}\gamma(\alpha+2,\beta R)}{(\frac{R}{\beta}\gamma(\alpha+1,\beta b) - \frac{1}{\beta^2}\gamma(\alpha+2,\beta b)) - (\frac{R}{\beta}\gamma(\alpha+1,\beta a) - \frac{1}{\beta^2}\gamma(\alpha+2,\beta a))}$$